Derivare le seguenti funzioni nella variabile t:

1. $y = a\cos\omega t.$ $\quad [y' = -a\omega \operatorname{sen}\omega t]$

2. $y = \cos t - \operatorname{sen}(a+t) - \operatorname{sen} a.$ $\quad \{y' = -[\operatorname{sen} t + \cos(a+t)]\}$

3. $y = At^2 \cdot e^{-\frac{E}{kt}}.$ $\quad \left[y' = Ae^{-\frac{E}{kt}}\left(2t + \frac{E}{k}\right)\right]$

4. $i = \frac{f}{R}\left(1 - e^{-\frac{tR}{L}}\right).$ $\quad \left[i' = \frac{f}{L}e^{-\frac{tR}{L}}\right]$

5. $i = I\cos(\omega t + \varphi).$ $\quad \left[\frac{di}{dt} = -I\omega \operatorname{sen}(\omega t + \varphi)\right]$

6. $s = s_0 + v_0 t.$ $\quad \left[\frac{ds}{dt} = v_0\right]$

7. $s = \frac{1}{2}at^2 + v_0 t + s_0.$ $\quad \left[\frac{ds}{dt} = at + v_0\right]$

8. $\Phi = BS\cos\omega t.$ $\quad \left[\frac{d\Phi}{dt} = -BS\omega \operatorname{sen}\omega t\right]$

9. $x = A \cdot e^{-\alpha t} \cdot \cos\omega t.$ $\quad \left[\frac{dx}{dt} = -Ae^{-\alpha t}(\alpha\cos\omega t + \omega \operatorname{sen}\omega t)\right]$

10. $y = -v_0 \operatorname{sen}\alpha t + \frac{1}{2}gt^2.$ $\quad \left[\frac{dy}{dt} = -\alpha v_0 \cos\alpha t + gt\right]$

11. $L = RI^2\frac{t}{2}.$ $\quad \left[\frac{dL}{dt} = \frac{1}{2}RI^2\right]$

12. $i = I\operatorname{sen}\left(\frac{2\pi t}{T} - \varphi\right).$ $\quad \left[\frac{di}{dt} = \frac{2\pi I}{T}\cos\left(\frac{2\pi t}{T} - \varphi\right)\right]$

Derivare le seguenti funzioni nella variabile ω:

13. $f = m\omega^2 r.$ $\quad \left[\frac{df}{d\omega} = 2m\omega r\right]$

14. $s = r\cos\omega t.$ $\quad \left[\frac{ds}{d\omega} = -rt\operatorname{sen}\omega t\right]$

15. $v = -r\omega \operatorname{sen}\omega t.$ $\quad \left[\frac{dv}{d\omega} = -r(\operatorname{sen}\omega t + \omega t\cos\omega t)\right]$

16. $F = \frac{1}{2} I\omega^2.$ $\left[\frac{dF}{d\omega} = I\omega\right]$

17. $E = \frac{m\omega^2 R^2}{4}.$ $\left[\frac{dE}{d\omega} = \frac{m\omega R^2}{2}\right]$

18. $z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$ $\left[\frac{dz}{d\omega} = \frac{\left(\omega L - \frac{1}{\omega C}\right)\left(L + \frac{1}{\omega^2 C}\right)}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\right]$

Derivare le seguenti funzioni nella variabile v:

19. $s = \frac{v^2 \operatorname{sen} 2\alpha}{g}.$ $\left[\frac{ds}{dv} = \frac{2 \operatorname{sen} 2\alpha}{g} v\right]$

20. $R = -\left(\frac{v^2}{l} + p \operatorname{sen} \alpha\right).$ $\left[\frac{dR}{dv} = -\frac{2}{l} v\right]$

21. $E = \frac{1}{2} m v^2.$ $\left[\frac{dE}{dv} = mv\right]$

22. $R = \frac{1}{T}\left(p + \frac{a}{v^2}\right)(v - b).$ $\left[\frac{dR}{dv} = \frac{1}{T}\left(\frac{2ab}{v^3} - \frac{a}{v^2} + p\right)\right]$

23. $y = e^{v^2 - \sqrt{2}k}(k + e^{-k}).$ $\left[\frac{dy}{dv} = 2v(k + e^{-k})e^{v^2 - \sqrt{2}k}\right]$

Verificare che le seguenti funzioni **soddisfano l'equazione** *a fianco indicata:*

1. $f(x) = e^{-x},$ $f(0) + xf'(0) = 1 - x.$

2. $f(x) = \sqrt{1 + x},$ $f(3) + (x - 3)f'(3) = 2 + \frac{x - 3}{4}.$

3. $f(x) = \operatorname{tg} x,$ $g(x) = \ln(1 - x).$ $\frac{f'(0)}{g'(0)} = -1.$

4. $f(x) = 1 - x,$ $g(x) = 1 - \operatorname{sen} \frac{\pi x}{2},$ $\frac{g'(1)}{f'(1)} = 0.$

5. $y = xe^{-x}$ $xy' = (1 - x)y.$

6. $y = \frac{1}{1 + x + \ln x},$ $xy' = y(y \ln x - 1).$

7. $y = ae^{-x},$ $y' + y = 0.$

8. $y = ae^{-2x} + \frac{5}{2},$ $y' + 2y = 5.$

9. $y = \frac{x - e^{-x^2}}{2x^2},$ $xy' + 2y = e^{-x^2}.$

10. $f(x) = \frac{\cos^2 x}{1 + \operatorname{sen}^2 x},$ $f\left(\frac{\pi}{4}\right) - 3f'\left(\frac{\pi}{4}\right) = 3.$

11. $y = (x^2 + 1)(e^x + c),$ $y' - \frac{2xy}{x^2 + 1} = e^x(x^2 + 1).$

12. $y = 2\cos x,$ $y^2 + y'^2 + \frac{4y^2}{y'^2} = 4 \operatorname{cosec}^2 x.$

13. $y = ae^{-2x} + \frac{1}{5}(\operatorname{sen} 4x - 2\cos 4x),$ $y' + 2y = 2 \operatorname{sen} 4x.$

14. $y = \ln \frac{1}{x+1}$, $\qquad xy' + 1 = e^y$.

15. $y = \ln x + ax + 1$, $\qquad xy' - y + \ln x = 0$.

16. $y = \frac{x^2}{2} + \frac{1}{2}x\sqrt{x^2+1} + \ln\sqrt{x + \sqrt{x^2+1}}$, $\qquad 2y = xy' + \ln y'$.

17. $y = \frac{\text{arcsen}\,x}{\sqrt{1-x^2}}$, $\qquad (1-x^2)y' - xy = 1$.

Derivate d'ordine superiore

1. *Calcolare le derivate prime e seconde delle seguenti funzioni:*

a) $y = 5x + 3$. $\quad [y' = 5;\ y'' = 0]$

b) $y = \frac{1}{2}x^2 + 9x - 2$. $\quad [y' = x + 9;\ y'' = 1]$

c) $y = 3x^4 - 2x^2 + 5x$. $\quad [y' = 12x^3 - 4x + 5;\ y'' = 36x^2 - 4]$

d) $y = \frac{1}{x}$. $\quad \left[y' = -\frac{1}{x^2};\ y'' = \frac{2}{x^3}\right]$

e) $y = x(x-1)^3$. $\quad [y' = (x-1)^2(4x-1);\ y'' = 6(x-1)(2x-1)]$

f) $y = x\ln x$. $\quad \left[y'' = \frac{1}{x}\right]$

g) $y = \frac{1}{\sqrt{1-x^2}}$. $\quad \left[y'' = \frac{1+2x^2}{\sqrt{(1-x^2)^5}}\right]$

2. *Calcolare la derivata* **seconda** *delle seguenti funzioni:*

a) $y = \frac{1}{4}x^2(2\ln x - 3)$. $\quad [y'' = \ln x]$

b) $y = e^{x^2}$. $\quad [y'' = e^{x^2}(4x^2 + 2)]$

c) $y = (1+x^2)\,\text{arctg}\,x$. $\quad \left[y'' = 2\,\text{arctg}\,x + \frac{2x}{1+x^2}\right]$

d) $y = \frac{1}{3}x^2\sqrt{1-x^2} + \frac{2}{3}\sqrt{1-x^2} + x \cdot \text{arcsen}\,x$. $\quad [y'' = 2\sqrt{1-x^2}]$

3. *Calcolare le derivate dei primi tre ordini delle seguenti funzioni:*

a) $y = \frac{x^2+1}{x^2-1}$. $\quad \left[y' = \frac{-4x}{(x^2-1)^2};\ y'' = \frac{12x^2+4}{(x^2-1)^3};\ y''' = \frac{-48x^3 - 48x}{(x^2-1)^4}\right]$

b) $y = \frac{\ln x}{x}$. $\quad \left[y' = \frac{1-\ln x}{x^2};\ y'' = \frac{2\ln x - 3}{x^3};\ y''' = \frac{11 - 6\ln x}{x^4}\right]$

c) $y = x\sqrt{1-x^2}$. $\quad \left[y' = \frac{1-2x^2}{\sqrt{1-x^2}};\ y'' = \frac{2x^3-3x}{\sqrt{(1-x^2)^3}};\ y''' = -\frac{3}{\sqrt{(1-x^2)^5}}\right]$

4. *Verificare che le seguenti funzioni* **soddisfano l'equazione** *a fianco indicata:*

a) $y = e^x\,\text{sen}\,x$, $\qquad y'' - 2y' + 2y = 0$.

b) $y = e^{-x}\,\text{sen}\,x$, $\qquad y'' + 2y' + 2y = 0$.

c) $y = \frac{x^2+2x+2}{2}$, $\qquad 1 + y'^2 = 2yy''$.

d) $y = \frac{x-3}{x+4}$, $\qquad 2y'^2 = (y-1)y''$.

) $y = \sqrt{2x - x^2}$, $\quad y^3 y'' + 1 = 0.$

) $y = \dfrac{1}{2} x^2 e^x$, $\quad y'' - 2y' + y = e^x.$

) $y = e^{4x} + 2e^{-x}$, $\quad y''' - 13y' - 12y = 0.$

) $y = c_1 e^{-x} + c_2 e^{-2x} (\forall c_1,\ c_2 \in \mathbb{R})$, $\quad y'' + 3y' + 2y = 0.$

) $y = e^{\sqrt{x}} + e^{-\sqrt{x}}$, $\quad xy'' + \dfrac{1}{2} y' - \dfrac{1}{4} y = 0.$

) $y = \operatorname{sen} e^x + \cos e^x$, $\quad y'' - y' + y e^{2x} = 0.$

n) $y = e^{2x} \operatorname{sen} 5x$, $\quad y'' - 4y' + 29y = 0.$

) $y = e^{-x} \cos x$, $\quad y^{(\mathrm{IV})} + 4y = 0.$

) $y = (x^2 - 1)^n$, $\quad (x^2 - 1) y^{(n+2)} + 2xy^{(n+1)} - n(n+1) y^{(n)} = 0.$