

Eseguire i seguenti **esercizi riassuntivi di derivazione**, dimostrando che si ha:

$$1. D \left[\frac{1}{15} \cos^3 x (3 \cos^2 x - 5) \right] = \operatorname{sen}^3 x \cos^2 x.$$

$$2. D \operatorname{tg} \frac{1 - e^x}{1 + e^x} = - \frac{2e^x}{(1 + e^x)^2} \operatorname{sec}^2 \left(\frac{1 - e^x}{1 + e^x} \right).$$

$$3. D \frac{\ln \operatorname{sen} x}{\ln \cos x} = \frac{\operatorname{ctg} x \ln \cos x + \operatorname{tg} x \ln \operatorname{sen} x}{\ln^2 \cos x}.$$

$$4. D(\operatorname{arcsen} x + \sqrt{1 - x^2}) = \sqrt{\frac{1 - x}{1 + x}}.$$

$$5. D \ln \frac{x + \sqrt{1 - x^2}}{x} = - \frac{1}{x \sqrt{1 - x^2} (x + \sqrt{1 - x^2})}.$$

$$6. D \operatorname{sen}^2 \left(\frac{1 - \ln x}{x} \right) = \frac{\ln x - 2}{x^2} \operatorname{sen} \left[2 \left(\frac{1 - \ln x}{x} \right) \right].$$

$$7. D[\ln(\sqrt{1 + e^x} - 1) - \ln(\sqrt{1 + e^x} + 1)] = \frac{1}{\sqrt{e^x + 1}}.$$

$$8. D \operatorname{sen} \sqrt{1 - 2^x} = - \frac{\cos \sqrt{1 - 2^x}}{2\sqrt{1 - 2^x}} 2^x \ln 2.$$

$$9. D \left[\frac{a}{2} \left(e^{\frac{x}{a}} - e^{-\frac{x}{a}} \right) \right] = \frac{1}{2} \left(e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right).$$

$$10. D a^{\operatorname{tg} nx} = n a^{\operatorname{tg} nx} \cdot \operatorname{sec}^2 nx \cdot \ln a.$$

$$11. D \ln \frac{1}{x + \sqrt{x^2 - 1}} = - \frac{1}{\sqrt{x^2 - 1}}.$$

$$12. D \sqrt{a \operatorname{sen}^2 x + b \cos^2 x} = \frac{(a - b) \operatorname{sen} 2x}{2\sqrt{a \operatorname{sen}^2 x + b \cos^2 x}}.$$

$$14. D \frac{\arccos x}{\sqrt{1 - x^2}} = \frac{x \arccos x - \sqrt{1 - x^2}}{\sqrt{(1 - x^2)^3}}.$$

$$13. D \operatorname{arcsen} \frac{x^2 - 1}{x^2} = \frac{2}{x \sqrt{2x^2 - 1}}.$$

$$16. D(\operatorname{arcsen} x^2 + \arccos x^2) = 0.$$

$$15. D \left[\frac{1}{2} \operatorname{ltg} \frac{x}{2} - \frac{1}{2} \frac{\cos x}{\operatorname{sen}^2 x} \right] = \frac{1}{\operatorname{sen}^3 x}.$$

$$17. D \left[\frac{1}{2} (\operatorname{arcsen} x)^2 \arccos x \right] = \frac{1}{2} \frac{\operatorname{arcsen} x (2 \arccos x - \operatorname{arcsen} x)}{\sqrt{1 - x^2}}.$$

19. $D[x - \ln(2e^x + 1 + \sqrt{e^{2x} + 4e^x + 1})] = \frac{1}{\sqrt{e^{2x} + 4e^x + 1}}$.
20. $D[\ln(x \operatorname{sen} x \sqrt{1-x^2})] = \frac{1}{x} - \frac{x}{1-x^2} + \operatorname{ctg} x$. 21. $D\sqrt[5]{(1+x e^{\sqrt{x}})^3} = \frac{3e^{\sqrt{x}}(2+\sqrt{x})}{10\sqrt[5]{(1+x e^{\sqrt{x}})^2}}$.
22. $D[\ln(e^x + 5 \operatorname{sen} x - 4 \operatorname{arcsen} x)] = \frac{(e^x + 5 \cos x) \sqrt{1-x^2} - 4}{(e^x + 5 \operatorname{sen} x - 4 \operatorname{arcsen} x) \sqrt{1-x^2}}$.
23. $D[\operatorname{arctg}(\ln x) + \ln(\operatorname{arctg} x)] = \frac{1}{(1+\ln^2 x) \cdot x} + \frac{1}{(1+x^2) \operatorname{arctg} x}$.
24. $D \operatorname{arcsen} \frac{x}{\sqrt{1+x^2}} = \frac{1}{1+x^2}$. 25. $D\left[\frac{1}{\sqrt{b}} \operatorname{arcsen}\left(x\sqrt{\frac{b}{a}}\right)\right] = \frac{1}{\sqrt{a-bx^2}}$.
26. $D\left(\sqrt{a^2-x^2} + a \cdot \operatorname{arcsen} \frac{x}{a}\right) = \sqrt{\frac{a-x}{a+x}}$. 27. $D\left(x\sqrt{a^2-x^2} + a^2 \cdot \operatorname{arcsen} \frac{x}{a}\right) = 2\sqrt{a^2-x^2}$.
28. $D\left(2 \operatorname{arcsen} \frac{x-2}{\sqrt{6}} - \sqrt{2+4x-x^2}\right) = \frac{x}{\sqrt{2+4x-x^2}}$.
29. $D \ln \cos\left(\operatorname{arctg} \frac{e^x - e^{-x}}{2}\right) = \frac{e^{-x} - e^x}{e^{-x} + e^x}$.
30. $D\left[\ln\sqrt[4]{\frac{x^2+x+1}{x^2-x+1}} + \frac{1}{2\sqrt{3}}\left(\operatorname{arctg} \frac{2x+1}{\sqrt{3}} + \operatorname{arctg} \frac{2x-1}{\sqrt{3}}\right)\right] = \frac{1}{x^4+x^2+1}$.
31. $D\left(\frac{x \operatorname{arcsen} x}{\sqrt{1-x^2}} + \ln\sqrt{1-x^2}\right) = \frac{\operatorname{arcsen} x}{(1-x^2)\sqrt{1-x^2}}$.
32. $D\left(\frac{\sqrt{2}}{3} \operatorname{arctg} \frac{x}{\sqrt{2}} + \frac{1}{6} \ln \frac{x-1}{x+1}\right) = \frac{x^2}{x^4+x^2-2}$.
33. $D\left[\ln\left(\frac{1+x}{1-x}\right)^{\frac{1}{4}} - \frac{1}{2} \operatorname{arctg} x\right] = \frac{x^2}{1-x^4}$. 34. $D\left[\operatorname{arctg} \frac{a}{x} + \ln\sqrt{\frac{x-a}{x+a}}\right] = \frac{2a^3}{x^4-a^4}$.
35. $D\left(\sqrt{a^2-x^2} + a \cdot \operatorname{arcsen} \frac{x}{a}\right) = \sqrt{\frac{a-x}{a+x}}$.
36. $D\left(x\sqrt{a^2-x^2} + a^2 \cdot \operatorname{arcsen} \frac{x}{a}\right) = 2\sqrt{a^2-x^2}$.
37. $D\left(\ln \operatorname{arcsen} x + \frac{1}{2} \ln^2 x + \operatorname{arcsen} \ln x\right) = \frac{1}{(\operatorname{arcsen} x) \sqrt{1-x^2}} + \frac{\ln x}{x} + \frac{1}{x \sqrt{1-\ln^2 x}}$.
38. $D[\ln(\sqrt{1+e^x} - 1) - \ln(\sqrt{1+e^x} + 1)] = \frac{1}{\sqrt{1+e^x}}$.
39. $D \frac{1}{\sqrt{3}} \ln \frac{\operatorname{tg} \frac{x}{2} + 2 - \sqrt{3}}{\operatorname{tg} \frac{x}{2} + 2 + \sqrt{3}} = \frac{1}{1+2 \operatorname{sen} x}$.